International Journal of Mathematics and
Computer Applications Research (IJMCAR)
ISSN(P 2249-6955; ISSN(E): 2249-8060
Vol. 5, Issue 4, Aug 2015, 93-100
TJPRC Pvt. Ltd.

# AN ORDER LEVEL INVENTORY MODEL FORPERISHABLE ITEMS AND VARIABLE DETERIORATION RATE WITH TRADE CREDIT

**RUCHIKATYAGI[1] & PRASHANTCHOUHAN[2]**

[1]Assistant Professor, KITPS College, Moradabad, Uttar Pradesh, India

[2]Assistant Professor, IFTM University, Moradabad, Uttar Pradesh, India

## ABSTRACT

In this paper, we study the classical inventory model is constant for deteriorating items with time-dependent demand and partial backlogging. We have discussed in this paper the first model for perishable items, that demand is not only a function of stock but also perish-ability rate as well as demand increases with time and fluctuates with time. In this paper the second model for equal to the cycle time for setting the account and the credit period is less than.

**KEYWORDS:** An Order Level Inventory Model Forperishable items and Variable Deterioration Rate with Trade Credit

## INTRODUCTION

Trade Credit (permissible delay in payment) is a widely used business strategy. To supplies trade credit helps to expand sale but it also adds to the risk of bad debts at the same time. To buyers, trade credit provides a very big advantage due to the fact that they do not have to pay the seller immediately after receiving the items, but instead can delay their payment unit the end of the allowed period. At present, there are three bind of trade credit, in the first case, the length of the credit period is fixed, in the second case, the length of the credit period is closely relative to the ordering quantity and in the third case, the sellers project trade credit to part of the ordering quantity and the purchase cost of the rest quantity should be paid immediately after receiving the items. Inflation and time value of money have also received attention of researchers. Several authors have worked on inventory models under trade credit policy.

Aggarwal and jaggi (1) also developed under ordering policies of deteriorating items under permissible delay in payments. Chang (5) presented model for the situation where the demand rate is a time continuous function and items deteriorate at a constant rate with partial backlogging. Sarbjit andShivraj (16) developed an inventory an inventory model having linear demand rate under permissible delay in payments with constant rate of deterioration. Khanra, Ghosh and chaudhuri (17) are considered An EOQ model for deteriorating item with time dependent quadratic demand under permissible delay in payment.

## NOTATIONS

$A = Ordering\ cost$

$C_D = Cost\ of\ deterioration$

$C_H = Holding\ cost\ due\ to\ deterioration\ of\ materials$

$\mathrm{P_T} = \text{The interest payable}$

$I_T = Interest\ earned$

$C_{VT} = Total\ variable\ cost\ per\ cycle$

$\mathrm{C_T(P, T) =}$ Variable cost per unit time

$I_t = Inventory\ at\ time\ t$

$T = Cycle\ period$

$M_T = Amount\ of\ items\ deteriorates\ during\ one\ cycle$

$\mathrm{t_1}$ = Is the time at which shortage starts, $0 \leq \mathrm{t_1} \leq \mathrm{T}$

## FORMULATION AND SOLUTION OF THE MODEL

**Model: 1**

The instantaneous inventory level $I(t)$at any time t during the cycle time t is governed by the following differential equation

$$\frac{\mathrm{DI(t)}}{\mathrm{dt}} + \mu\mathrm{tI(t)} = -(\mathrm{a} + \mathrm{mt}) \qquad 0 \leq \mathrm{t} \leq \mathrm{T} \tag{1}$$

Where P = $\mu$t and Q = - (a+mt)

The Solution of Equation (1) Is Given By

$$I(t)e^{\frac{\mu t^2}{2}} = -\int (a + mt)e^{\frac{\mu t^2}{2}}\, dt + C$$

$$= -\int (a + mt)\left[1 + \frac{\mu t^2}{2} + \cdots\right]$$

$$= -\int (a + mt)(1 + \frac{\mu t^2}{2})dt + C$$

$$= -\int a\, dt - \int \frac{a\mu t^2}{2}\, dt - \int mt\, dt - \int \frac{m\mu t^2}{2}\, dt$$

Expanding up to two terms

$$= -at - \frac{a\mu t^3}{6} - \frac{mt^2}{2} - \frac{m\mu t^4}{8}$$

$$= -a\left(t + \frac{\mu t^3}{6}\right) - m\left(\frac{t^2}{2} + \frac{\mu t^4}{8}\right) + C$$

At t = 0, $I(t) = I_0$ $\quad (Initial\ inventory\ level)$

Substituting this value in the above equation, we get

$$I_0 = C$$

Therefore

$$I(t) = I_0 e^{-\frac{\mu t^2}{2}} - a\left(t + \frac{\mu t^3}{6}\right)e^{-\frac{\mu t^2}{2}} - m\left(\frac{t^2}{2} + \frac{\mu t^4}{8}\right)e^{-\frac{\mu t^2}{2}} \qquad 0 \leq t \leq T \tag{2}$$

Now at t = T i.e. at the end of cycle period $I(t) = 0, so\ equation\ (2) becomes$

$$I_0 e^{-\frac{\mu t^2}{2}} - a\left(t + \frac{\mu t^3}{6}\right)e^{-\frac{\mu t^2}{2}} - m\left(\frac{t^2}{2} + \frac{\mu t^4}{8}\right)e^{-\frac{\mu t^2}{2}} = 0$$

$$I_0 = a\left(T+\frac{\mu T^3}{6}\right)+\left(\frac{T^2}{2}+\frac{\mu T^4}{8}\right) \tag{3}$$

Substituting the value of $I_0$ in equation (2) we get

$$I(t) = \left[a\left(T+\frac{\mu T^3}{6}\right)+m\left(\frac{T^2}{2}+\frac{\mu T^4}{8}\right)-a\left(t+\frac{\mu t^3}{6}\right)-m\left(\frac{t^2}{2}+\frac{\mu t^4}{8}\right)\right]e^{-\frac{\mu T^2}{2}}$$

$$= \left[\left\{a(T-t)+\frac{\mu}{6}(T^3-t^3)\right\}+m\left\{\left(\frac{T^2}{2}-\frac{t^2}{2}\right)+\frac{\mu}{8}(T^4-t^4)\right\}\right]e^{-\frac{\mu T^2}{2}} \qquad 0 \le t \le T \tag{4}$$

The Total Demand During Cycle Period T Is $\int_0^T (a+mt)dt$. Thus It Can Be Easily Seen That The Amount Of Items Deteriorates During One Cycle Is Given By:

$$D_T = I_0(\textit{initial inventory level} - \textit{total demand during cycle period } T)$$

$$= I_0 - \int_0^T (a+mt)dt$$

$$= a\left(T+\frac{\mu T^3}{6}\right)+m\left(\frac{T^2}{2}+\frac{\mu T^4}{8}\right)-\left(aT+\frac{mT^2}{2}\right)$$

$$D_T = a\left(\frac{\mu T^3}{6}\right)+\frac{m\mu T^4}{8} \tag{5}$$

On Ordering Cost Is Given By

$$C_0 = A$$

The cost of deterioration

$$C_D - CD_T = C\theta\left(\frac{aT^3}{6}+\frac{bT^4}{8}\right)$$

The holding cost during the period (0, T) is given by

$$C_H = I_C\int_0^T I(t)\ dt$$

$$= \mathrm{I_C}\left(\frac{\mathrm{aT^2}}{2}+\frac{\mathrm{a\mu T^2}}{12}+\frac{\mathrm{mT^3}}{3}+\frac{\mathrm{m\mu T^5}}{15}\right)\text{(neglecting higher power } \mu \text{ and } \mu \text{ is very small)}$$

The total cost of the uncertain inventory at time t is the cost of the current inventory at any time t, decrease the profit on the amount sold during time R, decrease the interest accepted from the sales per cycle during time R. Now, the interest payable every cycle for the inventory not being sold after due date is given by:

$$P_T = I_P\int_R^P [cI(t)-(s-c)\int_0^R (a+mt)dt - sI_c\int_0^R (a+mt)tdt]\ dt - (s-c)I_P\int_0^{P-R}(a+mt)tdt$$

$$= I_P\int_R^P cI(t)-(s-c)\left[at+\frac{mt^2}{2}\right]_0^R - sI_c\left[at+\frac{mt^2}{2}\right]_0^R-(s-c)I_P\left[\frac{at^2}{2}-\frac{mt^3}{3}\right]_0^{P-R}$$

$$= I_P c\,[a(P-R)T\left(1-\frac{\mu(P^2+R^2+PR)}{6}\right)-\frac{P+R}{12}[(6-\mu(P^2+R^2)+\frac{\mu T^3}{6}]$$

$$\mathrm{N}+m\{\frac{T^2}{2}P-R-\frac{P^3-R^3}{6}\left(\frac{T^2}{2}-1\right)+\frac{\mu(P^5-R^5)}{40}+\frac{\mu}{8}T^4(P-R)\}$$

$$-\left\{(s-c)\left(aR+\frac{mR^2}{2}+sI_c\left(\frac{aR^2}{2}+\frac{bR^3}{3}\right)\right\}(P-R)-(s-c)I_P\left(a\frac{(P-R)^2}{2}+m\frac{(P-R)^2}{3}\right)\right]$$

Increase interest every cycle, $I_T$ is the interest increase during the positive inventory and is given by

$$I_T = sI_C\left(\int_0^R (a+mt)t\,dt + \int_0^{T-P}(a+mt)tdt\right)$$

$$= sI_c\left[\left\{\frac{at^2}{2}+\frac{bt^3}{3}\right\}\begin{matrix}R\\0\end{matrix} + \left\{\frac{at^2}{2}+\frac{bt^3}{3}\right\}\begin{matrix}T-P\\0\end{matrix}\right]$$

$$= sI_c\left[\frac{a}{2}(R^2+(T-P)^2)+\frac{m}{3}(R^3+(T-P)^3)\right]$$

The Total Cost = Ordering Cost + Carrying Cost +Holding Cost+ Interest Cost + Payable Minus - Interest Earned

$$= C_O + C_D + C_H + P_T - I_T$$

$$= A + \frac{ca\mu T^3}{6} + +ic\left(\frac{aT^2}{2}+\frac{a\mu T^4}{12}+\frac{mT^3}{3}+\frac{m\mu T^5}{15}\right)$$

$$+I_P\ c\ \left[a\ (P-R)\left\{T\left(1-\frac{\mu(P^2+R^2+PR)}{6}\right)-\frac{P+R}{12}\left(6-\mu(P^2+R^2)\right)+\frac{\mu T^3}{6}\right\}\right]$$

$$m\ \left\{\frac{T^2}{2}(P-R)-\frac{P^3-R^3}{6}\left(\frac{T^2}{2}-1\right)+\frac{\mu(P^5-R^5)}{40}+\frac{\mu}{8}T^4(P-R)\right\}-\ (s-c)I_P$$

$$\left(\frac{a(P-R)^2}{2}+\frac{m(P-R)^3}{3}\right)-sI_c\left[\frac{a}{2}(R^2+(T-P)^2)+\frac{m}{3}(R^3+(T-P)^3)\right]$$

To Minimize Total Average Cost per Unit Time, the Optimal Values of Can Be Obtained By

$$\frac{\partial C_T}{\partial t} = 0$$

$$\frac{\partial C_T}{\partial T} = 0$$

Provided, They Satisfy the Following Conditions

$$\frac{\partial^2 C_T}{\partial t^2} > 0\ ,\frac{\partial^2 C_T}{\partial T^2} > 0$$

$$\left(\frac{\partial^2 C_T}{\partial t^2}\right)\left(\frac{\partial^2 C_T}{\partial T^2}\right)-\left(\frac{\partial^2 C_T}{\partial t\partial T}\right) > 0$$

## NUMERICAL EXAMPLE

A=Rs.200,c=40,s=46,$I_p$=0.18, $I_c$=0.15,$\mu$=0.20,P=1000,R=0,a=1unit,T=H/m,H=1

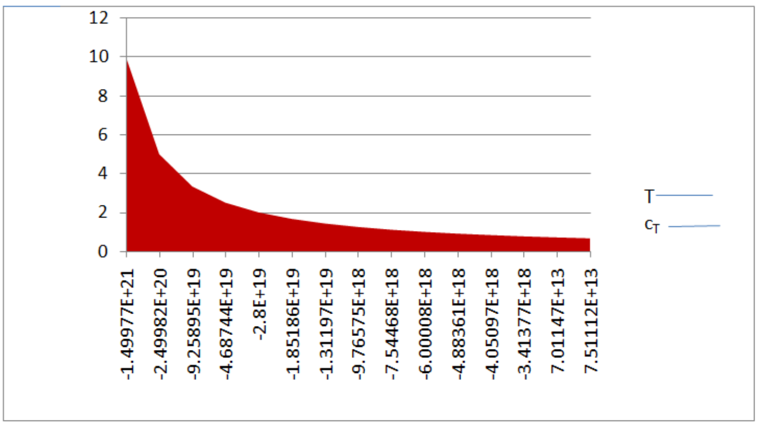


**Figure 1: Graphical Representation of Inventory System**

**Table 1**

| M | $P_T$ | $I_T$ | T | $C_T$ |
|---|---|---|---|---|
| 1 | 333873333 | -2228306355 | 10 | -1.49977E+21 |
| 2 | 667206667 | -4527928839 | 5 | -2.49982E+20 |
| 3 | 1.001E+09 | -6827802706 | 3.333333 | -9.25895E+19 |
| 4 | 1.334E+09 | -9127739585 | 2.5 | -4.68744E+19 |
| 5 | 1.667E+09 | -1.1428E+10 | 2 | -2.8E+19 |
| 6 | 2.001E+09 | -1.3728E+10 | 1.666667 | -1.85186E+19 |
| 7 | 2.334E+09 | -1.6028E+10 | 1.428571 | -1.31197E+19 |
| 8 | 2.667E+09 | -1.8328E+10 | 1.25 | -9.76575E+18 |
| 9 | 3.001E+09 | -2.0628E+10 | 1.111111 | -7.54468E+18 |
| 10 | 3.334E+09 | -2.2928E+10 | 1 | -6.00008E+18 |
| 11 | 3.667E+09 | -2.5228E+10 | 0.909091 | -4.88361E+18 |
| 12 | 4.001E+09 | -2.7528E+10 | 0.833333 | -4.05097E+18 |
| 13 | 4.334E+09 | -2.9828E+10 | 0.769231 | -3.41377E+18 |
| 14 | 4.667E+09 | -3.2128E+10 | 0.714286 | 7.01147E+13 |
| 15 | 5.001E+09 | -3.4428E+10 | 0.666667 | 7.51112E+13 |

**Model: 2**

The instantaneous inventory level $I(t)$ at any time t during the cycle time t is governed by the following differential equation

$$\frac{dI(t)}{dt} + \theta I(t) = -M(t)\ 0 \le t \le T \tag{1}$$

With the boundary conditions$I(0) = I_0, I(T) = 0\ and\ M(t) = Pe^{at}$

The Solution of Eq. (1) Is

$$I(t) = \frac{P}{a+\theta}\left[e^{(a+\theta)T-\theta t} - e^{at}\right], \quad 0 \le t \le T \tag{2}$$

If a= 0, then Eq. (2) represents the instantaneous inventory level at any time t for the constant demand rate.

The initial order quantity is

$$I_0 = I(0) = \frac{P}{a+\theta}\left[e^{(a+\theta)T} - 1\right] \tag{3}$$

The total demand during the cycle period [0, T] is

$$\int_0^T M(t)\,dt = \frac{P}{a}\left[e^{aT} - 1\right] \tag{4}$$

Number of deteriorating units is

$$I_0 - \int_0^T M(t)\,dt = P\left[\frac{1}{a+\theta}\left(e^{(a+\theta)T} - 1\right) - \frac{1}{a}\left(e^{aT} - 1\right)\right] \tag{5}$$

Deterioration cost for the cycle [0,T] is $g \times (number\ of\ deteriorating\ units)$

$$= gP\left[\frac{1}{a+\theta}\left(e^{(a+\theta)T} - 1\right) - \frac{1}{a}\left(e^{aT} - 1\right)\right] \tag{6}$$

Total Holding Cost for the Cycle [0, T] Is

HC = $h\ \int_0^T I(t)dt = \frac{Ph}{a+\theta}\left[\frac{1}{\theta}\left(e^{(a+\theta)T} - e^{aT}\right) - \frac{1}{a}\left(e^{aT} - 1\right)\right]$

Where $h = h_g g$ (7)

Since the interest is payable during the time $(T - t_1)$, the interest payable in any cycle [0, T] is

$$IQ_1 = gI_g \int_{t_1}^{T} I(t)\, dt = \frac{gI_gP}{a+\theta}\left[\frac{1}{\theta}e^{aT}\left(e^{\theta(T-t_1)} - 1\right) - \frac{1}{a}\left(e^{aT} - e^{at_1}\right)\right] \tag{8}$$

Interest earned in the cycle period [0, T] is

$$IR_1 = gI_e \int_{0}^{T} tM(t)dt = \frac{gI_eP}{a}\left[Te^{aT} - \frac{1}{a}\left(e^{aT} - 1\right)\right] \tag{9}$$

Total variable cost per cycle = replenishment cost + inventory holding cost + deterioration cost + interest Payable during the permissible delay period – interest earned during the Cycle.

The total variable cost per cycle per unit time is

$$C_{VT} = \frac{A}{T} + \frac{Ph}{T(a+\theta)}\left[\frac{1}{\theta}\left(e^{(a+\theta)T} - e^{aT}\right) - \frac{1}{a}\left(e^{aT} - 1\right)\right] + \frac{gP}{T}\left[\frac{1}{a+\theta}\left(e^{(a+\theta)T} - 1\right) - \frac{1}{a}\left(e^{aT} - 1\right)\right]$$

$$+ \frac{gI_gP}{T(a+\theta)}\left[\frac{1}{\theta}e^{aT}\left(e^{\theta(T-t_1)} - 1\right) - \frac{1}{a}\left(e^{aT} - e^{at_1}\right)\right] - \frac{gI_eP}{Ta}\left[Te^{aT} - \frac{1}{a}\left(e^{aT} - 1\right)\right] \tag{10}$$

To minimize total average cost per unit time, the optimal values of can be obtained by

$$\frac{dC_{VT}}{dT} = 0 \; and \; \frac{d^2C_{VT}}{dT^2} > 0$$

## NUMERICAL EXAMPLE

P= 1000 units per year, a= 1 unit per year, A= Rs.200 per order,$I_g$=0.15 per year, $I_e$=0.13 per year, h=0.12 per year, g=Rs.20 per unit, $\theta$=0.20 and $t_1$=0.15 year.Where the T=$t_1$

**Table 2**

| M | $C_H$ | $C_D$ | T | $C_{VT}$ |
|---|---|---|---|---|
| 1 | 162754.8 | 22026.47 | 10 | 717979611 |
| 2 | 403.4288 | 148.4132 | 5 | 3618217.8 |
| 3 | 54.59815 | 28.03162 | 3.333333 | 570388.58 |
| 4 | 20.08554 | 12.18249 | 2.5 | 207714.97 |
| 5 | 11.02318 | 7.389056 | 2 | 104045.92 |
| 6 | 7.389056 | 5.29449 | 1.666667 | 59894.353 |
| 7 | 5.552708 | 4.172734 | 1.428571 | 36229.208 |
| 8 | 4.481689 | 3.490343 | 1.25 | 21476.83 |
| 9 | 3.793668 | 3.037732 | 1.111111 | 11241.528 |
| 10 | 3.320117 | 2.718282 | 1 | 3554.3722 |
| 11 | 2.976979 | 2.482065 | 0.909091 | -2576.975 |
| 12 | 2.718282 | 2.300976 | 0.833333 | -7698.946 |
| 13 | 2.517023 | 2.158106 | 0.769231 | -12133.88 |
| 14 | 2.356418 | 2.042727 | 0.714286 | -16082.67 |
| 15 | 2.225541 | 1.947734 | 0.666667 | -19676.31 |

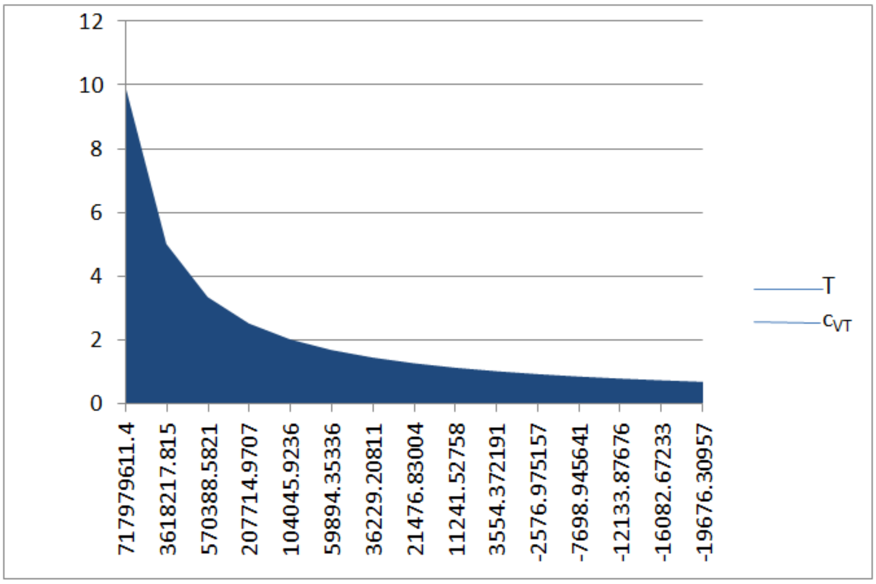


**Figure 2: Graphical Representation of Inventory System**

## RESULTS

We took two equations one is linear and other is exponential and applied same example for both equations and found there are no changes in result. The property of graph is increasing-steady-decreasing and becomes asymptotic.

## CONCLUSIONS

In this paper, we have discussed two models are given with different deterioration rate. In first model, deterioration is depending on time with stock depending demand, for items having quadratic demand, effect of inflation and considered the time value of money and is second model, weibull distribution is also considered. Shortages are available with partial backlogging. *Lead* Time is zero and which is a function of exponential decreasing function of time. This type of model is very useful for the items like computer chips and seasonal goods etc. As a result, constant deterioration rate to variable deterioration rate and time dependent deterioration.

## REFERENCES

1. Aggarwal, S.P. and C.K. Jaggi, 1995. Ordering policies of deteriorating items under permissible delay in payments. J. Operat. Res. Soc., 46: 658-662. http://www.jstor.org/pss/2584538.
2. Bierman, H. and J. Thomas, 1977. Inventory decisions under inflationary condition. Dec. Sci., 8: 151-155.
3. Buzacott, J.A., 1975. Economic order quantities with inflation.Operat. Res. Q., 26: 553-558.
4. Chandra, M.J. and M.L. Bahner, 1985. The effect of inflation and time valur of money on someinventory systems. Int. J. Prod. Res., 23: 723-729. http://cat.inist.fr/?aModele=afficheN&cpsidt=8515393 .
5. Chang, H.Y. and C.Y. Dye, 1999. An EOQ model for deteriorating items with time varying demand anpartialbacklogging.J.Operat.Res.Soc.,50:1176-1182http://cat.inist.fr/?aModele=afficheN&cpsidt=1993340.
6. Chung, K.J. and Y.F. Huang, 2004. Optimal replenishment policies for EOQ inventory model withLimited storage capacity under permissible delay in payments.Operat. Res., 41:

16-34.http://direct.bl.uk/bld/PlaceOrder.do?UIN=154981 078&ETOC=RN&from=search engine

7. Covert, R.P. and G.C. Philip, 1973. An EOQ model for items with weilbull distribution deterioration.IIETrans.,5:323326.http://www.informaworld.com/smpp/ftinterface?content=a780509777&rt=0&Format=pdf
8. Ghare, P.M. and G.F. Schrader, 1963. A model for exponentially decaying inventories. J. Ind. Eng., 14:238-43.
9. Goyal, S.K., 1985. Economic order quantity under conditions of permissible delay in payments. J. Operat. Res. Soc., 36: 335-338. http://www.jstor.org/stable/2582421.
10. Jamal, A.M.M., B.R. Sarkar and S. Wang, 1997. An ordering policy for deteriorating items withAllowable shortage and permissible delay in payment. J. Operat. Res. Soc., 48: 227-236.
11. Misra, R.B., 1979. A note on optimal inventory management under inflation. Naval Res. Logist., 26: 161-165.
12. Shah,Y.K.,1977.Anorderlevellotsizeinventorymodelfordeterioratingitems.IIETrans.,9:108http://www.informaworld.com/smpp/ftinterface?co ntent=a780520916&RT=0&format=pdf.
13. Sarbjit, S. and S.S. Raj, 2007. Lot sizing decisions under trade credit with variable demand rate underInflation. Ind. J. Math. Sci., 3: 29-38. http://www.serialspublications.com/.
14. Su, C.T., L.I. Tong and H.C. Liao, 1996. An inventory model under inflation for stock-dependentDemand consumption rate and exponential decay. Opsearch, 20: 99-106.
15. Teng, J.T., 2002. On the economic order quantity under conditions of permissible delay in payment. J. Operat. Res. Soc., 53: 915-918. http://www.jstor.org/stable/822919.
16. Chang, J.T., C.T. Chang and S.K. Goyal, 2005. Optimal Pricing and ordering policy under Permissible Delay in Payment. Int. J. Prod. Econ., 97: 121-129. DOI: 10.1016/j.ijpe.2004.04.010 333.
17. S. Khanra, S.K. Ghosh and K.S. Chaudhuri , 2011, An EOQ model for deteriorating item with time Dependent quadratic demand under permissible delay in payment, Applied Mathematics and Computation, 1-9.